An Interaction With Propagation Of Gravitational And Electromagnetic Waves In Plasma

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Abstract

We Know that the gravitational waves and electromagnetic waves are important as carriers of energy and information. This paper is deals with the study of the propagation and interaction of Gravitational waves in plasmas, with emphasis on nonlinear effects and applications within astrophysics. The physical systems are described by the Einstein-Maxwell-fluid equations or Einstein-Maxwell-Vlasov equations, when a kinetic treatment is required. The small amplitude and high-frequency approximation is employed for the gravitational waves, such that Perturbative techniques can be applied and space-time can be considered locally flat, with a gravitational radiation field superimposed on it. The gravitational waves give rise to coupling terms that have the structure of effective currents in the Maxwell equations and an effective gravitational force in the equation of motion for the plasma. The Einstein field equations describe the evolution of the gravitational waves, with the perturbed energy-momentum density of the plasma and the electromagnetic field as a source. The processes that are investigated are gravitational waves exciting electromagnetic waves in plasmas, altering the optical properties of plasmas and accelerating charged particles.

Keywords: Electromagnetic Waves in Plasma, Gravitational Wave, Electromagnetic Waves, Wave Interaction, Wave Propagation.

Introduction

Waves can be found in Nature in many forms; waves on the ocean surface, acoustic waves from a beautiful music piece reaching your ear, ancient electromagnetic waves from a distant star etc. Virtually all physical systems sustain some kind of waves. They are important as carriers of energy and information from one place to another. In some physical systems waves play a central role in the dynamics. In other systems the waves are only a by-product, but may then be important as carriers of valuable information for us to study and thereby gain increased understanding of the nature of the system. The effects that determine the evolution of a wave can be broadly grouped as:

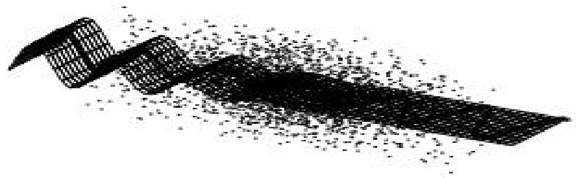
• Wave interactions
• Propagation in varying backgrounds
• Nonlinear self-interactions
• Wave instabilities

Wave interactions are not restricted to waves of the the same kind, but can also occur between waves of very different types. A striking example is the coupling between low-frequency waves in the earth's crust during earthquakes and electromagnetic waves in the ionosphere surrounding our planet. This can sometimes be observed as an illumination of the sky during an earthquake. Actually, it has been speculated whether observations of specific changes in the ionosphere can be a used to predict earthquakes. Waves may also interact directly with particles. A surfer (viewed as a particle) riding an ocean wave is such an example. The wave pushes (with the aid if gravity) the surfer forward, whereby the surfers velocity is increased. In so doing, some amount of energy is converted from wave energy to kinetic (particle) energy. If the number of particles (surfers) interacting with the wave is large, the interaction may lead to rapid damping of the wave and the particle system is "heated". The properties of a wave, such as speed, direction of propagation, energy, polarization etc, depends on the background in which it propagates. A background that varies in time and/or space leads to variations in these properties. This is what ”bends” light rays in a piece of glass and makes the stars ”twinkle” (or scintillate). The background in which the starlight propagates (e.g. the Earth’s atmosphere) fluctuates randomly, causing random variations in the intensity of the light. Nonlinear self-interaction of waves can be understood as follows. If a wave is powerful enough, it can affect the background in which it propagates. Since the evolution of the wave depends on the background, the wave thereby interacts with itself. For instance, a light beam may affect the background to make it behave like a focusing lens. As the light beam gets focused, the light intensity increases, leading to even larger effect on the background, i.e. stronger focusing, etc. The result is that the light beam ”collapses” into a narrow region with high energy density. Waves also function as a mathematical tool for investigating the stability of physical systems. By studying the equations that describe the surface of a lake, it is clear that it is a mathematically valid solution to have a surface as smooth as a mirror even on a windy day. If one, however, adds a very small perturbation (a small wave-ripple) to that solution one finds that the solution is unstable; the wind causes the small waves to grow larger. The conclusion is that — since there are always some very small wave-like irregularities even on a ”mirror-smooth” surface — if there is wind there will also be surface waves. This is an example of a wave instability. This paper is deals with study of gravitational and electromagnetic waves in plasmas, with main focus on how these waves can interact and on their propagation properties.

Gravitational and Electromagnetic Waves in Plasmas

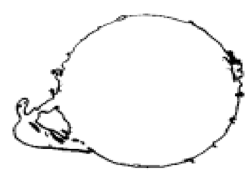
Gravitational waves are predicted to exist by the theory of general relativity but have, in the writing of this thesis, not yet been observed directly. Still, very few physicists doubt their existence. The reason is that general relativity has been very successful in explaining gravity; in the solar system, in astronomy and astrophysics, and in the evolution of the entire Universe starting with the Big Bang. If gravitational waves do not exist, the mentioned effects must be explained by a theory significantly different than relativity theory, and for this, there are no good candidates. The laws of 3 gravity that were written down by Isaac Newton in 1687 is a good description of most gravitational effects that occur on earth and in the solar system. In Newtonian gravity, massive objects act on each others with gravitational forces, according to Newtons laws. In general relativity, there are no gravitational forces. Gravity is described as curvature of space-time. In particular, a massive object (e.g. a star) curves the space-time. A light particle (e.g. a planet) tends to fall towards, or orbit, a more massive object. The explanation is, simply, that this is how objects move in curved space-time — not by being caused by a force acting on it, as in Newtonian theory. Space time is, however, not just an arena for objects to exist and interact in. It is in itself a dynamical object that possess energy, momentum and angular momentum. Gravitational waves can be described as ripples in the space time curvature. They propagate with the velocity of light in vacuum and carry energy, momentum and angular momentum, that can be transferred to particles and electromagnetic fields. In 1975, R. Hulse and J. Taylor reported on the discovery of a binary star system

(PSR 1913+16) that came to provide the so far strongest observational evidence for gravitational waves [1]. The two stars in the binary pulsar evolve around a common center of mass and have velocities as large as 0.1 percent of the speed of light. Observations show that the system loses energy — presumably by radiating gravitational waves — at precisely the rate predicted by relativity theory. There has been attempts since the 1960s to directly detect gravitational waves in earth-based laboratories, and the search intensified with the finding of the Hulse-Taylor pulsar. It is well understood why these attempts have so far been unable to register any gravitational waves. The reason is that gravitational waves interact very weakly with matter and the natural noise that exists in any detector (thermal vibrations, sound waves from the surroundings and even seismic disturbances) has typically a larger effect on the detector equipment than the gravitational waves. The technology has, however, progressed steadily and at the present time it is believed that current technology can reduce the noise and amplify the signal to a level where gravitational wave detection should be possible. One of the most promising detectors is LIGO [2], basically consisting of a system of laser beams and mirrors, that has been running since 2002 with continuously increasing sensitivity. The europeen detectors GEO600 [3] (technically less advanced) and VIRGO [4] (not yet in operation) uses the same technique. The detectors will later function as gravitational wave ”telescopes”. Astronomy today does not record only the visible light that reaches earth from cosmos. During the last decades a number of new ”windows to the universe” has been opened. These new windows consists of studying cosmic radio waves, micro waves, gamma rays, neutrinos etc. Each time a new observational window have been opened, astronomers have found new types of objects in space and learned of new Astrophyscial and cosmological processes. Most likely, the ”gravitational wave window” will be no exception. Gravitational wave astronomy is particularly interesting since gravitational waves are able to penetrate most barriers that hinder electromagnetic waves and particle rays from reaching our telescopes.



Fig(1). Illustration of the interaction between a gravitational wave (wavy surface), plasma particles (points) and an electromagnetic wave (solid curve). Although space-time is where plasmas and electromagnetic fields exist, it is also a dynamical object in itself. Energy and momentum can be transferred between space time, plasmas and electromagnetic fields.

To make sense of the images that will come from gravitational wave telescopes, it must first be understood how gravitational waves are produced and how they evolve as they propagate through space. Some waves are too weak to ever be detected on (or nearby) earth. They may, however, interact with matter and electromagnetic fields, as illustrated in Fig(1). and thereby give rise to secondary effects that potentially can be observed. In extremely energetic processes, such as supernovae explosions, gravitational waves interact more strongly and may be an important part of the dynamics. The interaction may also affect the gravitational wave and thus alter the form of the wave signals expected to reach the detectors. The common state of matter in these scenarios is the plasma state. A plasma is a collection of positively and negatively charged particles. This is the state that ordinary matter turns into if heated sufficiently, so that its molecular and atomic structure is disrupted. The plasma state remains as long as the kinetic energy of the particles is significantly larger than the attractive binding energy. At an early stage the entire universe was in a plasma state (except for the dark matter, possibly). In present time, the plasma state is still very common; stars are made of plasma and the ”empty” space between stars and galaxies is filled with a dilute plasma. Plasma physics has been stimulated mostly by the strive to understand the sun Fig(2), the earths magnetosphere and to construct fusion reactors. In many aspects a plasma behave like a fluid with electromagnetic properties. What distinguishes plasmas is the occurrence of collective behaviour and the variety of nonlinear effects. Collective behaviour is a result of the long-range electromagnetic interaction force, that tends to make particles move in a coordinated way. The nonlinearities leads to that motion and electromagnetic fields in plasmas tend to amplify each others (or their self, for that matter). Needless to say, perhaps, is that the behaviour of plasmas is in general rather complex and difficult to analyse. Much insight into the nature of plasmas can be gained, however, by studying the many type of wave phenomena that occur in plasmas.



Fig(2).The sun is, like most other plasmas, a rather lively system with many complex dynamical processes, e.g. solar eruptions that can be seen in the lower left of the picture. The nuclear burning in the center of the sun energizes the plasma. This is manisfested as turbulent motion, in which many nonlinear wave processes occur.

In plasmas one can find familiar wave types, like ”ordinary” electromagnetic waves and sound waves, but also waves that have no counterpart in other medias, and it is no exception in plasma physics, that the waves are important as carriers of energy and information.

General Relativistic Plasma Physics

General relativistic plasma physics is the interplay between space-time geometry, electromagnetic fields and matter that is in a plasma state. Each of these subfields have been well studied for, at least, almost a century. Still, this field is to a large extent unexplored, even though plasma is often the relevant state of matter where general relativistic effects are important. This is mainly because that both general relativity theory and plasma physics are both complex and highly nonlinear theories. To compensate for these difficulties, one often uses simplified models, e.g. simple matter models, like neutral ideal fluids. In many cases such simplifications are well justified, e.g. the gravitational coupling between two stars is not very sensitive to the internal structure of the stars. Describing the stars by ideal neutral fluids, or even as point particles, gives in many cases a sufficiently accurate result. In traditional plasma physics, the gravitational field from the plasma itself is usually discarded, since many electromagnetic effects dominate over gravitational effects. The occurrence of gravitational fields is usually due to some external source, e.g. a planet or a “neutral star”, and is described by Newton’s gravitational law. This can be generalized to non-gravitating plasmas in curved space-time, which may be relevant when considering relatively teneous plasmas in the vincinity of black holes and compact stars, or when considering the effect of gravitational waves on plasmas. When the plasma effect on the gravitational field is also of interest one should use a self-consistent description.

Governing equations

The elements of general relativistic plasma physics are: space-time geometry, electromagnetic fields and plasmas. The state of the space-time geometry — the gravitational field — is described by the metric tensor $g_{\mu\nu}$. The gravitational response to matter and electromagnetic fields is given by the Einstein field equations (EFE).

$$G_{\mu\nu} = \kappa T_{\mu\nu} \tag{1}$$

where $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$ is the Einstein tensor, $R_{\mu\nu}$ is the Ricci tensor, $T_{\mu\nu}$ is the total energy-momentum tensor for matter and electromagnetic fields,$\kappa = 8\pi G$ and G is the gravitational coupling constant. The twice contracted Bianchi identities, $\nabla_\nu G^{\mu\nu} = 0$, implies that also the total energy momentum tensor should be covariantly divergence free The electromagnetic field tensor, $F_{\mu\nu}$, is governed by the Maxwell equations

$$\nabla_\nu F^{\mu\nu} = j^\mu \tag{2}$$

$$\nabla_\mu F_{\nu\sigma} + \nabla_\nu F_{\sigma\mu} + \nabla_\sigma F_{\mu\nu} = 0 \tag{3}$$

Where j^μ is the total four-current. The energy-momentum tensor of the electromagnetic field is given by

$$T^{\mathrm{EM}}_{\mu\nu} = F_\mu^{\ \sigma} F_{\nu\sigma} - \tfrac{1}{4} g_{\mu\nu} F^{\sigma\tau} F_{\sigma\tau} \tag{4}$$

Treating the plasma as a collection of massive charged point-particles the plasma energy-momentum tensor is

$$T^{\mathrm{PL}}_{\mu\nu} = \frac{1}{\sqrt{|\det g_{\mu\nu}|}} \sum_{(i)} \frac{p^{(i)}_\mu p^{(i)}_\nu}{m\gamma} \delta^3(x^j - x^j_{(i)}) \tag{5}$$

where (i) labels the particles. The equations of motion for the particles are

$$p^\nu_{(i)} \nabla_\nu p^\mu_{(i)} = F^\mu_{(i)} \tag{6}$$

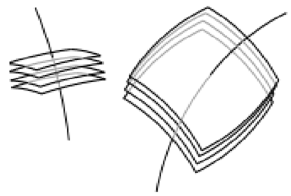
where $p^\mu_{(i)}$ is the four-momenta of particle (i) and $F^\mu_{(i)}$ is the four-force acting on it, e.g. the Lorentz force,

$$F^\mu_{(i)} = q_{(i)} p^\nu_{(i)} F_{\mu\nu}. \tag{7}$$

The governing equations presented are, unfortunately, not very practical for studying processes in general relativistic plasma physics. Firstly, the number of equations of motion for the plasma equals the number of particles. This makes the system intractable, even with the aid of powerful computers. Less complicated descriptions of the plasma are desirable and will be presented below. Secondly, with these formulations the physical nature of the system is not very transparent. For instance, real physical effects of the gravitational field on the plasma and electromagnetic field are mixed with “false” effects that can arise due to a particular choice of coordinates. In the following section, some methods for dealing with this type of problems are presented.

Unravelling The Physics

There are many ways to formulate the interplay between space-time and matter. Each alternative formulation has its own advantages and drawbacks. The generality, interpretability and simplicity varies with what mathematical framework one employs and what variables are studied. The approach presented here is based on the orthonormal frame formalism [11]. Space-time splitted into space and time Although the concept of space-time is central to general relativity it can often be practical to re-divide space-time into space and time. What an observer percepts as space is a hyper surface that is orthogonal to the (time like) four-velocity of the observer. The percieved direction of time coincides with that of the four-velocity. To be a useful tool, the splitting should not be restricted to one point (the observer) only. One can imagine space-time being covered by a field of ficticuous observers (or a physical field, e.g. a fluid velocity field), given by a time-like four-velocity field u^μ . Space time can then be sliced up into layers of surfaces (snapshots of space) — three-dimensional hyper surfaces, to be specific — that are orthogonal to u μ (direction of time), see Fig (3). This is often referred to as the 1+3 split



Fig(3). The 1+3 split illustrated. A field of observers with 4-velocities fills space time. At any point, space time is split into space (a hyper surface orthogonal to u^μ) and time (parallel to u^μ).

From the four-velocity u^μ one can construct the following projection tensors

$$U^\mu_\nu \equiv -u^\mu u_\nu \quad , \qquad H_{\mu\nu} \equiv g_{\mu\nu} + u_\mu u_\nu$$

$U^\mu{}_\nu$ projects parallel to u^μ and $H_{\mu\nu}$ projects onto a hyper plane orthogonal to u^μ . Note that

$$\delta^\mu_\nu = -u^\mu u_\nu + (\delta^\mu_\nu + u^\mu u_\nu) = U^\mu_\nu + H^\mu_\nu$$

Any vector f^μ can thus be split into a temporal part and a spatial part

$$f^\mu = U^\mu_\nu f^\nu + H^\mu_\nu f^\nu$$

The generalization to tensors is straight forward. Furthermore, it is convenient to introduce the rest-space volume element $\epsilon_{\mu\nu\sigma} \equiv u^\tau \epsilon_{\tau\mu\nu\sigma}.$ where

Example(1) (1+3 split of the electromagnetic field) Applying the projection tensors, the electromagnetic field can be split into an electric part, $E_\mu = F_{\mu\nu} u^\nu$, and a magnetic part,

$\bar{B}_\mu = \frac{1}{2}\epsilon_{\mu\nu\sigma} F^{\nu\sigma},$ (as measured by an observer with 4-velocity u^μ) by

$$F^{\mu\nu} = u^\mu E^\nu - u^\nu E^\mu + \epsilon^{\mu\nu\sigma} B_\sigma$$

The Maxwell equations can be spitted into a form that is rather similar to the Newtonian counterpart

$$H^\mu{}_\nu \dot{E}^\nu - \epsilon^{\mu\nu\sigma}\tilde{\nabla}_\nu B_\sigma = -J^\mu - \tfrac{2}{3}\Theta E^\mu + \sigma^\mu{}_\nu E^\nu + \epsilon^{\mu\nu\sigma}(\dot{u}_\nu B_\sigma + \omega_\nu E_\sigma) \qquad \text{(a)}$$

$$H^\mu{}_\nu \dot{B}^\nu + \epsilon^{\mu\nu\sigma}\tilde{\nabla}_\nu E_\sigma = -\tfrac{2}{3}\Theta B^\mu + \sigma^\mu{}_\nu B^\nu - \epsilon^{\mu\nu\sigma}(\dot{u}_\nu E_\sigma - \omega_\nu B_\sigma) \qquad \text{(b)}$$

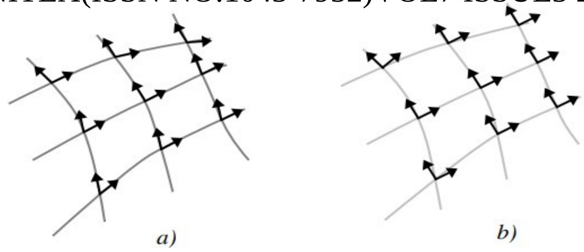
$$\tilde{\nabla}_\mu E^\mu = \rho_{\text{ch}} + 2\omega_\mu B^\mu \qquad \text{(c)}$$

$$\tilde{\nabla}_\mu B^\mu = -2\omega_\mu E^\mu \qquad \text{(d)}$$

The derivative operators that are introduced here and the kinematical quantities $\{\Theta, \sigma_{\mu\nu}, \omega_{\mu\nu}, \omega^\mu\}$ — referring to the state of the velocity field u^μ (and indirectly to the gravitational field) — are defined $J^\mu = H^\mu{}_\nu j^\nu$ and $\rho_{ch} = u_\mu j^\mu$ are the charge current and density, respectively. The Maxwell equations is this way splitted into two evolution equations, Eqs.(a) and (b), (Ampere's and Faraday's laws) and to constraint equations, Eqs. (c) and (d), (divergence of the electric and magnetic field), similar to their Newtonian counterparts.

Orthonormal Frame

In the coordinate formalism of general relativity the geometrical basis is in general not orthogonal, see Fig(4).By a coordinate transformation the basis can be made orthonormal locally — but a different transformation is in general required at another space-time point.



Fig(4). Illustration of coordinate basis versus orthonormal basis. The coordinate basis a) is tangential to the coordinate lines and is, in general, not orthogonal. Orthonormal basis b) can be constructed as linear combinations of the coordinate basis.

Gravitational and Electromagnetic waves

Gravitational waves

The following quotation is from a letter, written in 1936, from Albert Einstein to Max Born ”Together with a young collaborator, I arrived at the interesting result that gravitational waves do not exist, though they had been assumed a certainty to first approximation.” It is a remarkable statement since gravitational radiation was one of the first predictions (published in 1916 of Einstein’s general theory of relativity. But the prediction from 1916 was made for linear gravitational waves, i.e. a result of perturbation theory — which sometimes do fail. Later Einstein attempted to find exact wave solutions to the field equations, but the nonlinearities in the equations made it very difficult to find such solutions that are free from singularities. This lead him, and others, to believe that gravitational radiation is not a possibility. Although Einstein (and others) soon realized that the singularities where not as severe as they first seemed — and he returned to the belief that gravitational waves indeed could exist — gravitational waves remained a controverse until the 1970s. There are mainly two reasons for this. Firstly, even though the field equations implies that time-varying mass-quadrupole moments (such as binary stars).

emit gravitational waves, there was no sign of radiation reaction (change in energy and angular momentum) in the equation of motion for the source. The second reason is that the energy density carried by gravitational waves can, at any space-time point, be made zero by choosing a particular set of coordinates. The relativity community could not at that time agree on the meaning of this, i.e. if this means that gravitational waves are not real. But in the 1970s the reaction problem was finally well understood (for a review, see [34, 35]). Suitable formalisms for studying the equation of motion for binary stars had been found — the radiation reaction a raises first at fifth order in the post-Newtonian perturbation expansion. The discovery of the Hulse Taylor binary pulsar in 1975 [1], which looses energy at the predicted rate, came to provide the first observational evidence of gravitational waves. Still, the observed energy loss of the Hulse-Taylor pulsar can only be considered as an indirect observation of gravitational waves. The last decades, there has been much effort in designing detectors for direct observations of gravitational waves. These consist of large metal bars or spheres , or systems of lasers and mirrors (e.g. [2, 3, 4]) — as isolated from external and internal noise as current technology allows. It is believed that these detectors have now reached the level of sensitivity where it should be possible to detect some of the gravitational radiation bursts that Earth is occasionally exposed to, and the sensitivity is increased continuously. The rate of those events that should be possible to detect today are, however, expected to be low. Currently, the detector signals consist of what seem to be noise. Still, gravitational wave evidence may be hidden in these signals, which are analysed closely and compared to signals from other detectors. The analysis relies crucially on what the expected gravitational wave forms are. Although the first objective is to find evidence of gravitational radiation and thereby confirm this implication of general relativity, it is the next step that scientists find most exciting. The detectors will then function as gravitational wave ”telescopes”, see e.g. the review by Schutz. Gravitational radiation can penetrate most barriers that hinder conventional radiation from reaching earth. Gravitational wave telescopes will therefore open up a new window into space, through which scientist may be able to observe so far hidden regions, such as the interior of supernova explosions, emissions from neutron stars and black holes or the Big Bang. Another important feature is that gravitational waves come from the bulk motion of their sources, whereas most electromagnetic waves that are observed resides from individual particles. The most important sources of gravitational waves are considered.

•**Binary systems.** Two stars orbiting a common center of mass will radiate gravitational waves, leading to increasing orbital frequency and decreasing separation. For large separations, the waves are nearly monochromatic. If the stars are compact, e.g. neutron stars or black holes, the separation/frequency can eventually become small/high and the waves are then of larger amplitude and are modulated, before the objects finally coalesce. The orbital frequency ranges up to 1 kHz, before two neutron stars are in contact and begin merging.

•**Gravitational collapse**. Stars more massive than three solar masses are too heavy to cool down quietly when the nuclear burning ends. Instead they undergo gravitational collapse that ends with a supernova explosion, where huge amounts of energy is released mainly in the form of neutrinos but also in the form of gravitational waves, unless the collapse is spherically symmetric. The process is complex and it is unclear to what fraction energy is converted into gravitational waves. Frequencies in the kHz region are expected.

•**Neutron stars.** Newly formed neutron stars may rotate very fast (about 100 Hz). This has been shown to be an unstable state, due to the coupling between the neutron star normal modes and gravitational waves.The state of high rotation decays by the emission of gravitational waves.

•**Black holes.** If a black hole is perturbed, e.g. as an initial state after gravitational collapse, by annihilating a star or by merging with another black hole, its normal modes [42] are excited. These modes are associated with gravitational wave emission, see, e.g., the pedagogical review by Rezzolla. For stellar-mass black holes, the natural frequencies are of the order of 1 kHz.

•**The early Universe**. During the inflation of the universe, fluctuations in the gravitational wave field would have been energized by parametric amplification, resulting in a cosmic gravitational background radiation containing information of the state of the universe at times 10−24 s after the Big Bang. This should be compared to the scientific importance of the cosmic electromagnetic microwave background, which originated from times 105 years after Big Bang. Today, the spectrum of relic gravitational waves extends up to frequencies of GHz. But the background is stochastic, however, and best hope of detecting it is by placing detectors in space — operating in the frequency range mHz to Hz. Gravitational waves in plasma-like media have been studied since the 1970s. The studies can be categorised as follows. i) Observable effects on plasma. Some gravitational waves are too weak to ever be detected on earth using the current detector designs. Closer to the source, however, the waves are stronger and induce effects on plasmas

that may be observable — reaching earth electromagnetically or as particle showers. The time-delay of electromagnetic waves from pulsars due to gravitational waves has been proposed as a mean of observing gravitational waves. The acceleration of particles by gravitational waves in magnetized plasmas has been considered and the gravitational wave induced shift in the electromagnetic cyclotron spectrum was . Excitation of plasma Langmuir waves, plasma oscillations , ion-acoustic waves and electromagnetic waves by gravitational waves in plasmas have been studied. The conversion of gravitational waves into electromagnetic waves due to scattering against charged particles was calculated.

ii) Effect on gravitational waves. When propagating through a plasma-like medium, gravitational waves may exhibit dispersion, refraction, damping (absorption) and modification by interaction with other waves. Since the analysis of the signals from current gravitational wave detectors depends crucially on the expected wave-forms, it is necessary to determine the importance of these effects. Dispersion and possible Landau damping of gravitational waves in a collision less gas have been considered. Damping of gravitational waves due to resonant particle acceleration in magnetized plasma has been studied. whereas damping due to collisional dissipation was considered and in a cosmological fluid, the propagation of gravitational waves in matter was treated in terms of the electro-gravitational and magneto gravitational fields. A WKB formalism for gravitational wave propagation in a fluid medium was presented.The linearized Einstein-Vlasov equations have been solved for long gravitational waves in an ultra relativistic two-component Friedmann universe. The modification of gravitational waves due to cosmic magnetic fields was studied.Equations for nonlinear gravitational waves in matter have been derived.

iii) Gravitational wave emission. Plasma processes can also lead to the emission of gravitational waves. For the gravitational waves to be of significant amplitude, however, this requires very energetic plasma configurations. The generation of gravitational waves due to cosmic magnetic fields in the early universe was considered. The excitation of gravitational waves by interacting sound waves was studied and the conversion of electromagnetic waves into gravitational waves by scattering against charged particles was calculated.

iv) Interaction in complex processes. Many of the results in the points above can be important sub processes in larger, more complex, processes. This field is not very developed so far. The role of interaction between gravitational and electromagnetic waves in plasmas during supernova explosions was discussed. The conversion from gravitational waves to electromagnetic waves in plasmas has been studied in the context of gamma ray bursts in the coupling between gravitational waves and electromagnetic fields was proposed as a generating mechanism for the cosmic background magnetic field, and the implications of gravitational waves exciting plasma waves and electromagnetic waves in the cosmological plasma have been studied or discussed.

Linearized waves in vacuum

Most gravitational wave sources, e.g. compact binaries, are also associated with strongly curved background space-times, like the Schwarzchild space time. In general, it is difficult to distinguish the wave from the background. If the wavelength is much smaller than the characteristic length-scale of the background curvature, however, this distinction simplifies. This is referred to as the high-frequency approximation, for which Isaacson presented a formalism, where gravitational waves in curved space-time can be treated in a perturbative manor (up to third order in wave amplitude) . The idea, illustrated in Fig(5), is that in the limit of high frequencies, the background does not vary over distances comparable to the wavelength. By a coordinate transformation, the background space-time can then be made flat in any region of this size. When propagating over distances comparable to the background length scale, the background curvature makes the waves follow null geodesics and be ”deflected”, just like light rays. By taking into account higher order terms in the high-frequency approximation, the dispersion of gravitational waves by the background curvature is found.

Fig(5). Illustration of the high-frequency approximation. If the wavelength of the wave (wavy line) is much smaller than the distance over which the background space-time varies (curved line), the background can, locally, be taken to be flat (straight line).

Linearized Waves in a Plasma

The interaction of gravitational waves with plasmas and electromagnetic fields can be studied in two ways. Firstly, the plasma (and EM fields) can be treated as a test-plasma. The self-gravitation of the plasma is then discarded, and there is no back-raction of the plasma on the gravitational wave in this approximation. Secondly, the interaction can be examined using a self consistent description, where also the plasma contribution to the energy-momentum tensor in the EFE is taken into account. It is then possible to determine to what extent the gravitational waves are affected by the plasma.

Non-Gravitating plasma

In the case of gravitational waves in a test-plasma, plane gravitational waves can be taken to be in the TT-gauge, provided the wavelength and interaction region is small compared to the characteristic background length scale. The effect of gravitational waves on a plasma and an electromagnetic field is described by the equations. The calculations of these effects are simplified substantially by the TT-gauge choice. The effective gravitational force in the fluid description is given by

$$\begin{aligned}\mathbf{g} = &-\tfrac{1}{2}\gamma(1-v_z)\left[v_x\dot{h}_+ + v_y\dot{h}_\times\right]\mathbf{e}_1\\ &-\tfrac{1}{2}\gamma(1-v_z)\left[v_x\dot{h}_\times - v_y\dot{h}_+\right]\mathbf{e}_2\\ &-\tfrac{1}{2}\gamma\left[(v_x^2-v_y^2)\dot{h}_+ + 2v_xv_y\dot{h}_\times\right]\mathbf{e}_3 \qquad \text{(A)}\end{aligned}$$

and the gravitationally induced effective currents in the Maxwell equations are

$$\begin{aligned}\mathbf{j}_E = &-\tfrac{1}{2}\left[(E_x-B_y)\dot{h}_+ + (E_y+B_x)\dot{h}_\times\right]\mathbf{e}_1\\ &-\tfrac{1}{2}\left[-(E_y+B_x)\dot{h}_+ + (E_x-B_y)\dot{h}_\times\right]\mathbf{e}_2 \qquad \text{(B)}\end{aligned}$$

$$\begin{aligned}\mathbf{j}_B = &-\tfrac{1}{2}\left[(E_y+B_x)\dot{h}_+ - (E_x-B_y)\dot{h}_\times\right]\mathbf{e}_1\\ &-\tfrac{1}{2}\left[(E_x-B_y)\dot{h}_+ + (E_y+B_x)\dot{h}_\times\right]\mathbf{e}_2 \qquad \text{(C)}\end{aligned}$$

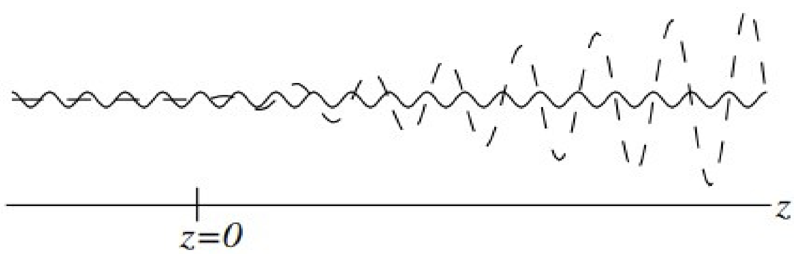
where the overdot represents partial time derivative and$\partial z \approx -\partial t$ has been used. The remaining gravitationally induced terms are zero. The effective force for particles, G, takes the same form as g: G = mg upon replacing v→p/γm. The effect of gravitational waves on matter and electromagnetic fields is illustrated by two examples below.

Example(2) (Acceleration of particles) In Example (1), the effect of gravitational waves on free particles was considered. A particle moving in a monochromatic gravitational wave-field will have a periodic shift in kinetic energy and direction of propagation, but for free particles (free from forces other than the gravitational), there is no net effect. If the particle is not free, however, and the motion is constrained somehow, this can lead to a cumulative change in energy and momentum, i.e. lead to a long term acceleration (or deceleration). Constrained motion occurs, for instance, for a charged particle in a magnetic field. Take the magnetic field to be B = Bez and assume that a gravitational wave propagates parallel to it. The equation of motion for the charged particle is then given by

$$\frac{d}{dt}\mathbf{p} = (\omega_c/\gamma)\mathbf{p}\times\mathbf{e_z} - \mathbf{G} \equiv \mathbf{F}$$

where$\omega_c \equiv qB/m$ is the cyclotron frequency. In the absence of the gravitational wave,**G = 0**, the particle gyrates around the magnetic field lines. A gravitational wave will in general produce a small and irregular deviation from the particle gyration. But if the wave is resonant with the particle gyration, which occurs if the wave frequency equals$\omega = 2\omega_c$, the z-component of the effective force, g_z, will contain a constant term that will lead to a long term acceleration of the particle along the magnetic field. the particle velocity approaches the speed of light as[1]

$$v_z \rightarrow 1 - \frac{1}{h\omega_c t}$$



Fig(7). Generation of an electromagnetic wave by a gravitational wave enetering a region (z > 0) with a magnetic field. The amplitude of the electromagnetic wave grows linearly with z.

where h denotes the gravitational wave amplitude. The size of the acceleration clearly depends on the strength of the wave and of the magnetic field. Given long time enough this mechanism can clearly produce highly energetic particles. It is crucial, however, that the magnetic field is sufficiently static and homogeneous for the resonance to be fulfilled for a long enough time. The acceleration or deceleration of particles means that the wave looses or gains some energy. The effect on the amplitude of gravitational waves, that interact resonantly with particles in magnetized plasmas, is considered.

Electromagnetic waves

In this paper, electromagnetic waves refer to waves in plasmas with E, B 6= 0. This includes, besides waves that resemble electromagnetic waves in vacuum, also low-frequency waves in magnetized plasmas, such as Alfven waves. In cosmological and astrophysical plasmas, electromagnetic waves is the main carrier of information. Many astrophysical plasmas, such as stars and interstellar plasmas, are well described by magnet hydro dynamics, where the electromagnetic waves are also very important in the dynamical processes. In laboratory plasmas, electromagnetic waves are for example applied for heating of fusion plasmas, as a diagnostic tool and to accelerate particles. Troughout this section, we discard gravitational fields.

Linearized waves

In linearized plasma wave theory [13]-[17], the wave perturbations are assumed to be of small amplitude. The precise conditions for the nonlinear terms to be negligible vary between different models and wave modes. In the fluid descriptions, the linearized governing equations — the Maxwell and plasma equations — are of the form **D^u = 0** (a1)

where $\hat{D} = \hat{D}(\partial t, \nabla)$ is a matrix of partial differential operators and u is a vector of wave variables (the electromagnetic field, the velocity field, density etc.). If the plasma is subjected to some external influence, this can be included as a source term, a vector S, on the right hand side of Eq.(a1). There are many ways of studying waves in plasmas. If the characteristic length scale of the wave perturbation is of the same order as of the background (or conversely for the time scales) the wave evolution can be rather complicated. In the limit of short wave lengths (and short time scales), on the other hand, the background can be taken as homogeneous (and stationary) and the analysis simplifies largely. The general solution can be constructed as a superposition of plane waves (or, more formally, by making a Fourier transformation)

$$\mathbf{u} = \sum_{\mathbf{k},\omega} \mathbf{u}_{\mathbf{k},\omega} e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}$$

where k is the wave vector,ωthe frequency. Since the $u_{k,\omega}$ factors are independent, Eq.(a1) implies $D_{k,\omega} u_{k,\omega} = 0$, where $D_{k,\omega}$ can be obtained from the operator matrix, Dˆ, by letting
$\partial_t \to -i\omega, \nabla \to ik$. For non-trivial solutions to exist, the wave vector and frequency should be consitent with $\det(D_{k,\omega}) = 0$. The determinant can in general be factorized
where $D_n(\omega, k)$ are polynomials inωand k. Each solution $D_n(\omega, k) = 0$ — referred to as a dispersion relation — is a distinct wave mode and corresponds to distinct state vectors $u_{k,\omega,n}$. The distinct wave modes can also be described by their wave equations. These can be obtained directly from Eq.(a1), by noting that some components form closed sets of equations, where each set can be combined into a single equation, or by taking$-i\omega \to \partial t, ik \to \nabla$, $u_{k,\omega,n} \to u_n$ to form, $\mathbf{\hat{D}_n u_n = 0}$

Wave propagation and interaction

Geometric optics

Geometric optics is an approximation method for describing propagation of linear waves in weakly varying backgrounds, i.e. when the characteristic wave length/frequency is small/large compared to the background length/time scale. The method is well described in Refs. [13] and covariantly in [22]. Consider a simple system

$\hat{D}(\partial t, \nabla, t, x)A = 0$ **(b1)**

where t and x in the wave propagator Dˆ denotes that the background is (weakly) t and x dependent. The wave propagator could for instance be that of Example with a varying plasma frequency,ω_p. Take the amplitude A to be of the form
$A = a(x,t)e^{i\theta(x,t)}$
and define wave number and frequency in terms of the wave phase (or eikonal)θ, as
$k = \nabla\theta, \omega = -\partial_t\theta$
Due to the weakly varying background, k andωwill also vary — they are the local wave number and frequency of the wave. It should be pointed out that this method applies both for single wave pulses as well as for quasi monochromatic long waves. The wave equation**(b1)** implies a local dispersion relation $D_{k,\omega}$ that is rewritten as$\mathbf{\omega = W(k, x,t)}$.With this method it is practical to conceptually treat the wave as a collection of quasi-particles— photons, if the wave is electromagnetic. The number density of photons is given by E/~ω, where E is the wave energy density (typically $E \propto |a(x,t)|^2$). Each photon has the equation of motion

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{v}_g \quad \text{(b2)}$$

where x(t) is the position and v_g the group velocity$\partial_k W$. The photon wave number and frequency evolves according to

$$\frac{d\mathbf{k}}{dt} = -\nabla W \ , \quad \frac{d\omega}{dt} = \partial_t W \quad \text{(b3)}$$

with the co-moving derivative defined as $\frac{d}{dt} \equiv \partial_t + \mathbf{v}_g \cdot \nabla$

Equations **(b2)** and **(b3)** are known as the ray equations. Note the Hamiltonian form of these equations, with caconical variables x and k, and Hamiltonian W. Clearly, in a stationary uniform media, the photon wave number as well as frequency is preserved in the co-moving frame. For a modulated wave, the local group velocity vary with the position in the pulse, and this give rise to pulse dispersion. In a stationary but non uniform media, the photon frequency remains unchanged whereas the wave number varies, and the photon is refracted. In a non-stationary media, the photon frequency is up or down shifted along the ray trajectories, a phenomena also known as photon acceleration.

Wave-Wave Interactions

In systems with many degrees of freedom, like plasmas, there are many ways that waves may interact with each other. The effect is most efficient when the interaction is coherent1 . In linear theory, interaction

between two waves, represented by ψ_1 and ψ_2, has the following form $\hat{D}_1\psi_1 = c_1\psi_2$, $\hat{D}_2\psi_2 = c_2\psi_1$ **(c1)**
where $\hat{D}_n$ is the wave propagator and c_n is a constant (for n = 1, 2). The two waves are coupled if $c_n \neq 0$, i.e. if they act as sources to each other. This picture is only meaningful if the coupling is weak. More specifically we assume c_n is small such that the waves can be represented by $\psi_n = \tilde{\psi}_n(x,t)e^{i(k_n\cdot x-\omega_n t)}$. The amplitude, $\tilde{\psi}_n(x,t)$, depends weakly on x and t on the scales k^{-1}_n and ω^{-1}_n , and k_n and ω_n satisfies the dispersion relation D_n. In that case, Eq. **(c1)** implies simple evolution equations for the wave amplitudes $\tilde{\psi}_n(x,t)$. Coherent interaction occurs when the frequencies and wave numbers coincide, $\omega_1 = \omega_2$ and $k_1 = k_2$. If this is not fulfilled (exactly or approximately), the interaction only produces small rapid variations in the wave amplitudes. Coherent interaction between gravitational and electromagnetic waves is considered, Two different wave modes with the same frequency rarely have the same wave number, as they fulfill different dispersion relations. But in non uniform media, the wave number changes with the varying background parameters during propagation, and there may exist critical points where the wave numbers coincide. At these points the waves become more strongly coupled (they interact coherently) and may undergo mode conversion. In nonlinear wave theory, there is a multitude of wave couplings that can occur. For a wave pulse
$\psi = \tilde{\psi}(x,t)e^{i(k\cdot x-\omega t)} + c.c.$
(where c.c. denotes complex conjugate of the preceeding term), quadratic nonlinearities give rise to terms of the form $|\tilde{\psi}|^2$, $\tilde{\psi}^2 e^{i2(k\cdot x-\omega t)}$ and $\tilde{\psi}^{*2} e^{-i2(k\cdot x-\omega t)}$ and similarly for cubic and higher order nonlinearities. These nonlinearities may act as source terms for other wave modes — providing coupling between waves that do not couple in linear theory.

Conclusion

Gravitational wave energy can be transferred to a plasma by exciting waves in the plasma. The process is most efficient if it is resonant. This can be analysed using the three-wave interaction formalism. In this paper, the decay of gravitational waves to magneto hydrodynamic waves through three-wave interaction is considered. As a first step a magneto hydrodynamic plasma model that takes space-time curvature into account is derived. The space-time is then taken to be that of linear gravitational waves in Minkowski space. The gravitational waves are assumed to propagate parallel to the plasma background magnetic field and the coupling to magneto hydrodynamic waves with arbitrary direction of propagation is considered. Coupled mode equations, that describe the evolution of the wave amplitudes due to three-wave interaction, are derived. The equations are shown to be energy conserving and fulfill the Manley-Rowe relations. Particular attention is paid to the case when the gravitational wave act as pump wave — having constant amplitude on the time scale of interest. In this case the magneto hydrodynamic waves undergo parametric excitation and their amplitudes grow exponentially in time, with the growth rate $\Gamma = \mathrm{SQRT}(C_I C^*_{II} h)$, where CI and CII are the coupling coefficients, appearing in the coupled mode equations, and h is the gravitational wave amplitude. In an idealized model of a magnetized plasma close to a compact binary, the exponential growth rate becomes $\Gamma \sim 10^{-2}\, s^{-1}$.

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